# A Mathematical Description of BJT Behavior

Now that we understand the **physical** behavior of a BJT—that is, the behavior for each of the three BJT **modes** (active, saturation, and cutoff)—we need to determine also the **mathematical** description of BJT behavior.

We will find that BJT behavior is in many was **similar** to MOSFET behavior!

## ACTIVE MODE

We found earlier that forward biasing the **emitter-base** junction (EBJ) results in **collector** (drift) current. The junction voltage for the EBJ is $v_{BE}$ (for npn).

Thus, in active mode, the voltage **base**-to-**emitter** $v_{BE}$ controls the **collector** current $i_C$. Specifically, we find that:

$$i_C = I_S \, e^{v_{BE}/V_T} \quad (npn)$$

$$i_C = I_S \, e^{v_{EB}/V_T} \quad (pnp)$$

Here we should note **two** things:

1. *The active mode equation is very* ***similar*** *to the* p-n *junction diode equation.*

No surprise here! The collector current is directly proportional to the **diffusion** current across the EBJ. That's why the equation is just like the diffusion current equation for a ***pn* junction**.

In fact, $I_S$ is **scale current** (a device parameter), and $V_T$ is the **thermal voltage** (25 mV)—the same values used to describe junction diodes!

2. *A BJT in ACTIVE mode is* ***analogous*** *to a MOSFET in SATURATION mode.*

Recall that for a MOSFET in SATURATION, the **drain** current $i_D$ is "**controlled**" by the **gate**-to-**source** voltage $v_{GS}$.

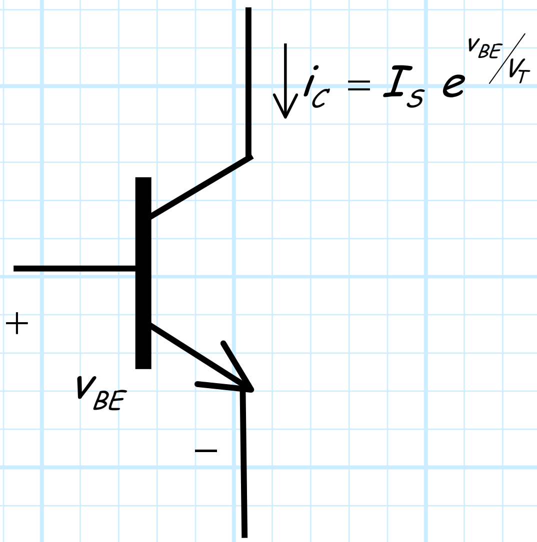
Likewise, for a BJT in ACTIVE mode, the **collector** current $i_C$ is "**controlled**" by the **base**-to-**emitter** voltage $v_{BE}$.

Note the **analogies**!

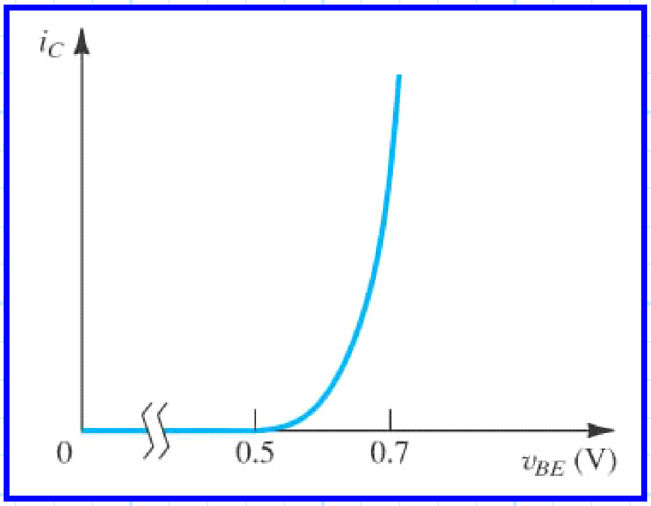
$i_D$ analogous to $i_C$

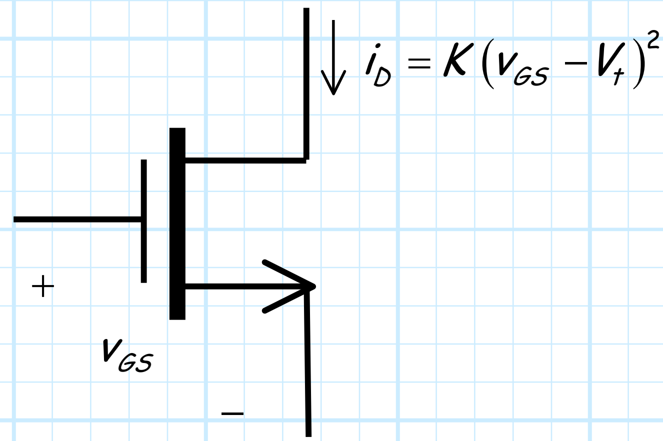
$v_{BE}$ analogous to $v_{GS}$

ACTIVE analogous to SATURATION

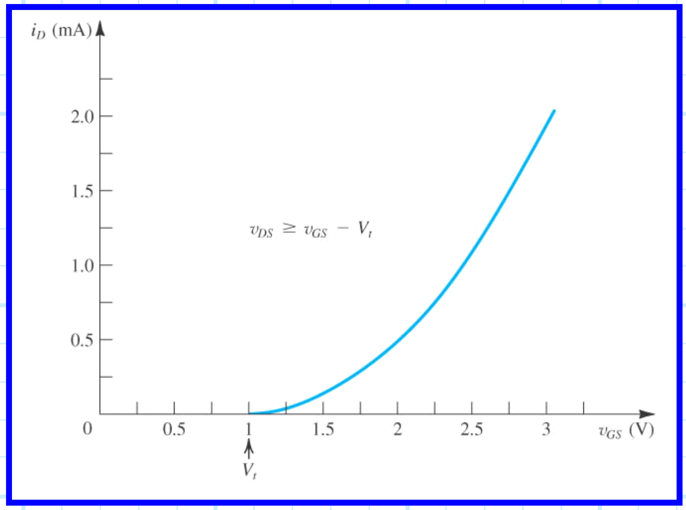



*npn* in ACTIVE mode




*NMOS* in SATURATION mode




Note also that a **necessary** (but not sufficient) condition for a *npn* BJT to be in ACTIVE mode is that $v_{BE} > 0$ (i.e., the **EBJ** is **forward biased**).

This is **analogous** to an NMOS in SATURATION, where a **necessary** (but not sufficient) condition is that $v_{GS} > V_t$ (i.e., the channel is conducting).

Likewise, for a BJT to be in the ACTIVE mode, the **CBJ** must be in **reverse bias** (i.e, $v_{BC}$<0). Assuming that the forward biased EBJ results in $v_{BE} \approx 0.7\text{ V}$, we can use **KVL** to determine that the CBJ will be reverse biased only when:

$$v_{CE} > 0.7\text{ V} \quad \text{for } npn \text{ in ACTIVE}$$

$$v_{EC} > 0.7\text{ V} \quad \text{for } pnp \text{ in ACTIVE}$$

These statements above are **analogous** to the MOSFET inequality $v_{DS} > v_{GS} - V_t$ for MOSFET SAT. (more on this later!).

Now, we are tempted to make **another analogy** between base **current** $i_B$ and gate **current** $i_G$, but here the analogies **end**!

Recall $i_G$=0 **always**, but for BJTs we find that $i_B$ is **not equal to zero** (generally).

Instead, we found that although **most** of the charge carriers (e.g., holes or free electrons) diffusing across the EBJ end up "**drifting**" across the CBJ into the **collector**, **some** charge carriers do "exit" the **base** terminal.

Recall, however, that for every **one** charge carrier that leaves the **base** terminal, there are typically **50 to 250** (depending on the BJT) charge carriers that drift into the collector.

As a result, the **collector current** for ACTIVE mode is typically 50 to 250 times **larger** than the **base current**! I.E.:

$$50 < \frac{i_C}{i_B} < 250 \quad \text{typically, for BJT ACTIVE}$$

The **precise** value of this ratio is the **device parameter** β (**beta**) :

$$\beta \doteq \frac{i_C}{i_B} \qquad \text{for BJT ACTIVE mode}$$

Thus, we find that the **base current** can be expressed as:

$$i_B = \frac{i_C}{\beta} = \frac{I_S}{\beta}\, e^{v_{BE}/V_T} \qquad (npn)$$

$$i_B = \frac{i_C}{\beta} = \frac{I_S}{\beta}\, e^{v_{EB}/V_T} \qquad (pnp)$$

Likewise, from **KCL**, we can determine the **emitter current** for a BJT in the ACTIVE mode:

$$\begin{aligned} i_E &= i_C + i_B \\ &= \beta\, i_B + i_B \\ &= (\beta + 1)\, i_B \end{aligned}$$

Or similarly,

$$\begin{aligned} i_E &= i_C + i_B \\ &= i_C + \frac{i_C}{\beta} \\ &= \left(1 + \frac{1}{\beta}\right) i_C \\ &= \left(\frac{\beta + 1}{\beta}\right) i_C \end{aligned}$$

An **alternative** to device parameter $\beta$ is the **device parameter** $\alpha$, defined as:

$$\alpha = \frac{\beta}{\beta + 1}$$

Note that the value of $\alpha$ will be just **slightly less than one**.

We can thus **alternatively** express the current relationships as:

$$i_C = \alpha\, i_E \qquad\qquad i_B = (1 - \alpha)\, i_E$$

And therefore:

$$i_E = \frac{i_C}{\alpha} = \frac{I_S}{\alpha}\, e^{v_{BE}/V_T} \quad (npn)$$

$$i_E = \frac{i_C}{\alpha} = \frac{I_S}{\alpha}\, e^{v_{EB}/V_T} \quad (pnp)$$

Recall that the **exponential** expression for a *pn* junction turned out to be of **limited** use, as it typically led to unsolvable **transcendental equations.**

The **same** is true for **these** exponential equations! We will thus generally use the equations below to **approximate** the behavior of a BJT in the ACTIVE mode:

$$v_{BE} \approx 0.7 \quad i_C = \beta\, i_B \quad v_{CE} > 0.7 \quad (npn \text{ in ACTIVE})$$

$$v_{EB} \approx 0.7 \quad i_C = \beta\, i_B \quad v_{EC} > 0.7 \quad (pnp \text{ in ACTIVE})$$

## SATURATION MODE

Recall for BJT SATURATION mode that **both** the CBJ and the EBJ are **forward biased**.

Thus, the collector current is due to **two** physical mechanisms, the **first** being charge carriers (holes or free-electrons) that

**drift** across the CBJ (just like ACTIVE mode), and the **second** being charge carriers that **diffuse** across the forward biased CBJ!

As a result, a **second term** appears in our mathematical description of **collector current** (when the BJT is in SATURATION):

$$i_C = I_S \, e^{v_{BE}/V_T} - \left(\frac{I_S}{\alpha_R}\right) e^{v_{BC}/V_T} \quad (npn)$$

$$i_C = I_S \, e^{v_{EB}/V_T} - \left(\frac{I_S}{\alpha_R}\right) e^{v_{CB}/V_T} \quad (pnp)$$

where $\alpha_R$ represents the **same** device parameter $\alpha$ discussed earlier (for ACTIVE mode), with the only difference that it specifies the value of $\alpha$ specifically for the **CBJ**.

This second term describes the current due to **diffusion** across the CBJ. Note that this current is in the **opposite** direction of the drift current (the first term), hence the **minus** sign in the second term.

Now using **KVL** (i.e., $v_{CE} = v_{CB} + v_{BE}$), we can write this collector current equation as:

$$i_C = I_S\, e^{v_{BE}/V_T} - \left(\frac{I_S}{\alpha_R}\right) e^{(v_{BE} - v_{CE})/V_T}$$

$$= I_S\, e^{v_{BE}/V_T} \left(1 - \frac{e^{-v_{CE}/V_T}}{\alpha_R}\right)$$

Thus, we can conclude:

$$i_C = I_S\, e^{v_{BE}/V_T} \left(1 - \frac{e^{-v_{CE}/V_T}}{\alpha_R}\right) \qquad \text{for } npn \text{ in SAT.}$$

$$i_C = I_S\, e^{v_{EB}/V_T} \left(1 - \frac{e^{-v_{EC}/V_T}}{\alpha_R}\right) \qquad \text{for } pnp \text{ in SAT.}$$

It is thus clear that for a BJT in SATURATION, the collector current $i_C$ is dependent on **both** $v_{BE}$ and $v_{CE}$.

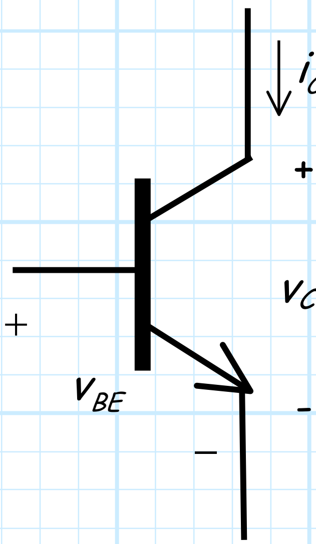
This is precisely **analogous** to the TRIODE mode for MOSFETS!

Recall for **triode** mode, drain current $i_D$ is dependent on both $v_{GS}$ and $v_{DS}$. We thus have discovered **two** new analogies:

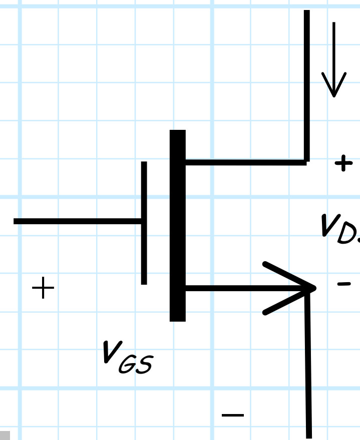
$v_{CE}$ analogous to $v_{DS}$

SATURATION analogous to TRIODE

$$i_C = I_S\, e^{v_{BE}/V_T}\left(1 - \frac{e^{-v_{CE}/V_T}}{\alpha_R}\right)$$

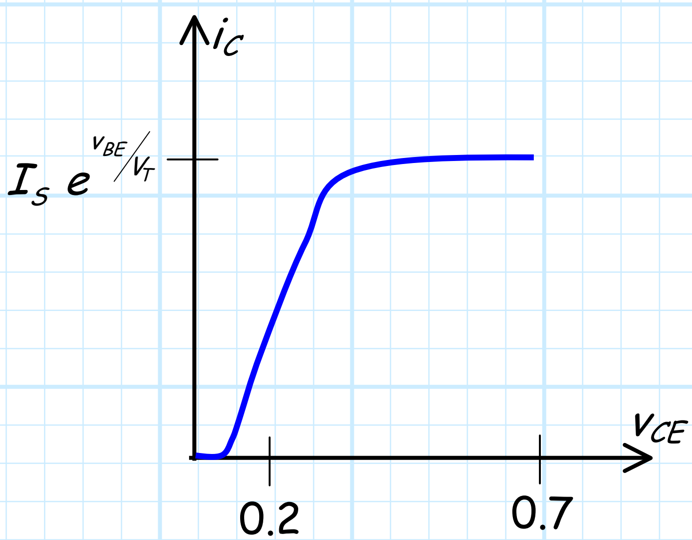


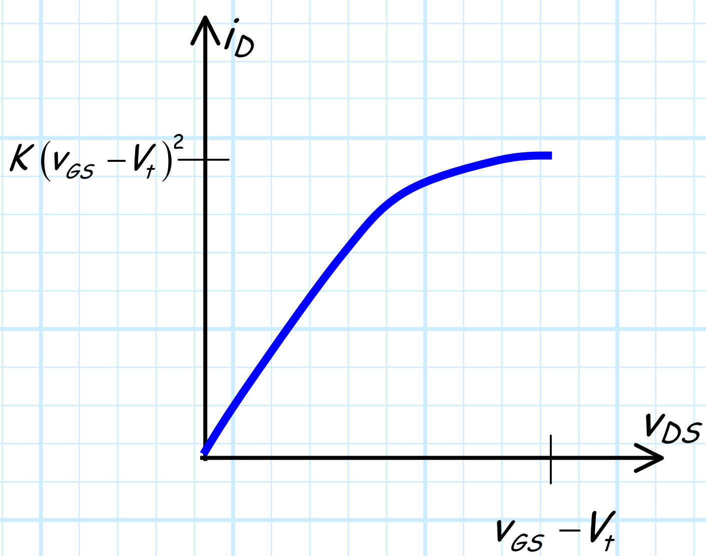

$$i_D = K\left[2\left(v_{GS} - V_t\right)v_{DS} - v_{DS}^2\right]$$




*npn* in SAT. mode

*NMOS* in TRIODE mode



Now, a BJT is in SATURATION mode if **both** the CBJ and the EBJ are **forward biased**. Assuming that $v_{BE} \approx 0.7\,V$ if the EBJ is forward biased, the CBJ voltage $v_{BC}$ will be positive **only** if (using **KVL**):

$$\begin{aligned} v_{BC} &> 0 \\ v_{BE} - v_{CE} &> 0 \\ 0.7 - v_{CE} &> 0 \\ v_{CE} &< 0.7 \end{aligned}$$

Thus, we can conclude that a **necessary** (but not sufficient) condition for a BJT to be in SATURATION is:

$$v_{CE} < 0.7 \qquad \text{for } npn \text{ in SAT.}$$

$$v_{EC} < 0.7 \qquad \text{for } pnp \text{ in SAT.}$$

These inequalities are **analogous** to the MOSFET inequalities:

$$v_{DS} < v_{GS} - V_t \quad \text{for NMOS in Triode}$$

$$v_{DS} > v_{GS} - V_t \quad \text{for PMOS in Triode}$$

Now, we note for the BJT SATURATION mode that the **collector current** will **always** be **less** that that in ACTIVE mode with the same value of $v_{BE}$:

Sat. $i_c$ Active $i_c$

$$I_S\, e^{v_{BE}/V_T}\left(1-\frac{e^{-v_{CE}/V_T}}{\alpha_R}\right) < I_S\, e^{v_{BE}/V_T} \quad \text{for all } v_{CE}$$

Thus, we can **equivalently** state that the collector current in SATURATION will be **less** than the value $\beta\, i_B$:

$$i_C < \beta\, i_B \quad \text{for BJT in SAT.}$$

This of course means that the **base** current in SAT. is **greater** than $i_C/\beta$ (i.e., the base current in active) :

$$i_B > \frac{i_C}{\beta} \quad \text{for BJT in SAT.}$$

Likewise, this means that:

$$i_E < (\beta+1)\, i_B \quad \text{and} \quad i_C < \alpha\, i_E \quad \text{for BJT in SAT.}$$

But remember KCL is still valid for BJTs in SATURATION (it's **always** valid!):

$$i_E = i_B + i_C \qquad \text{(KCL)}$$

Finally, we should again note that the **exponential** equations presented for SATURATION mode are **not** particularly useful for analyzing BJT circuits (that **transcendental** equation thing again!).

Thus, we describe a BJT in SATURATION with some **approximate** equations. Since both CBJ and EBJ are forward biased, we assume that $v_{BE} \approx 0.7V$ and that $v_{BC} \approx 0.5V$, resulting in the following **approximate** description for a BJT in SATURATION:

$$v_{BE} \approx 0.7\text{ V} \qquad v_{CE} \approx 0.2\text{ V} \qquad i_C < \beta i_B \qquad \text{for } npn \text{ in SAT.}$$

$$v_{EB} \approx 0.7\text{ V} \qquad v_{EC} \approx 0.2\text{ V} \qquad i_C < \beta i_B \qquad \text{for } pnp \text{ in SAT.}$$

## ***CUTOFF MODE***

Cutoff mode for BJTs is obviously **analogous** to cutoff mode for MOSFETS.

In both cases the transistor currents are **zero**!

$$i_E = i_B = i_C = 0 \qquad \text{for BJTs in CUTOFF}$$

Note that a BJT is in cutoff if **both** EBJ and CBJ are in **reverse bias**. This is true if:

$$v_{BE} < 0 \quad \text{and} \quad v_{BC} < 0 \qquad \text{for } npn \text{ in CUTOFF}$$

$$v_{EB} < 0 \quad \text{and} \quad v_{CB} < 0 \qquad \text{for } pnp \text{ in CUTOFF}$$